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PREDICTION OF MOTIONS OF AN AIRPLANE RESULTING FROM ABRUPT MOVEMENT OF LATERAL OR DIRECTIONAL CONTROLS

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PREDICTION OF MOTIONS OF AN AIRPLANE RESULTING FROM ABRUPT MOVEMENT OF LATERAL OR DIRECTIONAL CONTROLS

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SUMMARY

A procedure is presented for determining the motions of an airplane resulting from the deflection of the lateral or directional controls for the case of nonlinear derivatives. The step-by-step integration on which the procedure is based considers the rolling, the yawing, and the lateral accelerations computed from wind-tunnel data as functions of the sideslip angle. A sample computation table is presented to illustrate the application of the procedure.

A comparison is made of different methods for calculating the disturbed motions of an airplane resulting from an abrupt aileron movement. Experimental data, which were obtained from conventional wind-tunnel tests of a model of a recent fighter airplane, are used in the computations for comparing the various methods.

The resulting solutions show that, for the case of nonlinear derivatives, the calculated motions are in better agreement with the results obtained from flight tests if the rolling and yawing accelerations computed from static-model tests are considered as functions of the sideslip angle. The lateral acceleration, which is often assumed to be negligible, should be considered. The variation of the rolling and yawing accelerations resulting from aileron movement probably should also be considered when sufficient data are available. The variation of the dynamic derivatives L_p, N_p, L_r, and N_r should also be taken into account when sufficient dynamic-test data are available.

It is shown that the present step-by-step integration method is reliable for cases in which only the first quarter-cycle of the motion is required (for example, in cases in which the maximum value of the sideslip angle is desired for determining vertical-tail loads in rolling pull-outs). For the range past the first quarter-cycle of the motion curve, the method requires further refinements

such as those provided by the Runge-Kutta summation method. The present step-by-step integration method may be applied to the solution of motions produced by rudder movements or by a combination of rudder and aileron movement, as well as to the solution of motions produced by ailerons alone.

INTRODUCTION

The increasing importance of predicting the flying qualities and maneuverability of an airplane has emphasized the need for a more accurate method of computing the lateral motion resulting from abrupt control movement. Increased speed and maneuverability have, in addition, made it necessary to predict the maximum sideslip angles in lateral-control maneuvers in order that maximum vertical-tail loads may be estimated.

Much work has been done on the subject of disturbed motions (references 1 to 5), but all the solutions deal with constant lateral-stability derivatives. These treatments assume that the rolling-moment coefficient C_l and the yawing-moment coefficient C_n are linear functions of the sideslip angle β, the rolling velocity p, and the yawing velocity r. Wind-tunnel tests, however, indicate that most present-day airplanes do not possess these linear variations of C_l and C_n with β, since the degree of linearity is affected by such factors as the geometry of the airplane, the power, the type of propeller, and the blade angle.

Lack of mathematical equations for expressing the derivatives as functions of the motions makes the method of references 2 and 4 inapplicable. The procedure for the solution with nonlinear characteristics presented herein is a refinement and an expansion of the integration procedure of reference 1.

With the wind-tunnel data available at the present time, only the linear and angular accelerations βY_β, βL_β, βN_β, δL_δ, and δN_δ may be determined as functions of the sideslip angle β. Lack of model-test data for effects of the rate of roll p and the rate of yaw r still makes it necessary to deal with the dynamic derivatives L_p, L_r, and N_p determined from theoretical treatments (reference 3). The dynamic derivative N_r is determined partly from wind-tunnel data and partly from theoretical considerations (references 6 and 7).

In the present report three previously established procedures, based upon constant derivatives, for determining the disturbed motions of an airplane that result from abrupt aileron movement are compared with a step-by-step integration procedure that considers accelerations, computed from wind-tunnel data, as functions of the sideslip angle β. This step-by-step integration not only generally provides more accurate solutions for disturbed motions but also should prove useful in determining the vertical-tail loads resulting from rolling pull-out maneuvers as discussed in reference 5.

Unpublished experimental data (fig. 1) obtained from conventional wind-tunnel tests of a model of a recent fighter airplane are used in calculating the motions, and the results are compared with flight results.

COEFFICIENTS AND SYMBOLS

The coefficients and symbols used herein are referred to a system of axes in which the Z-axis is in the plane of symmetry and perpendicular to the relative air stream, the X-axis is in the plane of symmetry and perpendicular to the Z-axis, and the Y-axis is perpendicular to the plane of symmetry. The coefficients and symbols are defined as follows:

C_L airplane lift coefficient $\left(\frac{\text{Lift}}{qS}\right)$

C_{L_w} lift coefficient of wing

ΔC_{L_f} increment of lift coefficient resulting from flap deflection

$C_{d_{o_w}}$ profile-drag coefficient of wing

$\Delta C_{d_{o_f}}$ increment of profile-drag coefficient caused by flap deflection

C_l rolling-moment coefficient $\left(\frac{\text{Rolling moment}}{qSb}\right)$

C_n yawing-moment coefficient $\left(\frac{\text{Yawing moment}}{qSb}\right)$

C_{l_a} rolling-moment coefficient caused by aileron deflection

C_{n_a} yawing-moment coefficient caused by aileron deflection

C_Y lateral-force coefficient $\left(\frac{\text{Lateral force}}{qS}\right)$

b wing span, feet

b_f flap span, feet

λ taper ratio; ratio of tip chord to root chord

A aspect ratio

l distance from center of gravity to rudder hinge line, feet

δ_a aileron deflection, degrees; used with subscripts L and R to refer to left and right ailerons, respectively

δ_f flap deflection, degrees

δ_r rudder deflection, degrees

α_v angle of attack of vertical tail, degrees

α_a absolute angle of attack of wing measured from zero-lift line, degrees

ψ angle of yaw, degrees

β sideslip angle, radians except as otherwise indicated; considered in static wind-tunnel tests to be equal to $-\psi$

$C_{n_{\delta_r}}$ rate of change of yawing-moment coefficient with rudder deflection $\left(\frac{\partial C_n}{\partial \delta_r}\right)$

$\delta_{r_{\alpha_v}}$ inverse of rudder effectiveness parameter at constant lift $\left(\left(\frac{\partial \delta_r}{\partial \alpha_v}\right)_{C_L}\right)$

C_{l_p} rate of change of rolling-moment coefficient with wing-tip helix angle $\left(\dfrac{\partial C_l}{\partial \frac{pb}{2V}}\right)$

C_{n_p} rate of change of yawing-moment coefficient with wing-tip helix angle $\left(\dfrac{\partial C_n}{\partial \frac{pb}{2V}}\right)$

C_{l_r} rate of change of rolling-moment coefficient with $\dfrac{rb}{2V}$ $\left(\dfrac{\partial C_l}{\partial \frac{rb}{2V}}\right)$

C_{n_r} rate of change of yawing-moment coefficient with $\dfrac{rb}{2V}$ $\left(\dfrac{\partial C_n}{\partial \frac{rb}{2V}}\right)$

C_{n_ψ} rate of change of yawing-moment coefficient with angle of yaw $\left(\dfrac{\partial C_n}{\partial \psi}\right)$

L_p rate of change of rolling acceleration with rate of roll $\left(C_{l_p} \dfrac{b}{2V} \dfrac{qSb}{mk_X{}^2}\right)$

L_r rate of change of rolling acceleration with rate of yaw $\left(C_{l_r} \dfrac{b}{2V} \dfrac{qSb}{mk_X{}^2}\right)$

N_p rate of change of yawing acceleration with rate of roll $\left(C_{n_p} \dfrac{b}{2V} \dfrac{qSb}{mk_Z{}^2}\right)$

N_r rate of change of yawing acceleration with rate of yaw $\left(C_{n_r} \dfrac{b}{2V} \dfrac{qSb}{mk_Z{}^2}\right)$

δL_δ rolling acceleration caused by control deflection, radians per second per second $\left(\frac{C_l qSb}{mk_X^2}\right)$ (Subscripts a and r indicate aileron and rudder, respectively.)

δN_δ yawing acceleration caused by control deflection, radians per second per second $\left(\frac{C_n qSb}{mk_Z^2}\right)$ (Subscripts a and r indicate aileron and rudder, respectively.)

βL_β rolling acceleration resulting from sideslip angle, radians per second per second $\left(C_l \frac{qSb}{mk_X^2}\right)$

βN_β yawing acceleration resulting from sideslip angle, radians per second per second $\left(C_n \frac{qSb}{mk_Z^2}\right)$

βY_β sideslipping acceleration resulting from sideslip angle, feet per second per second $\left(C_Y \frac{qS}{m}\right)$

$\frac{dp}{dt}$ rolling angular acceleration, radians per second per second

$\frac{dr}{dt}$ yawing angular acceleration, radians per second per second

$\frac{d\beta}{dt}$ sideslipping velocity, radians per second

$\frac{dv}{dt}$ sideslipping acceleration, feet per second per second

ΣL_n net induced rolling accelerations at $t = n$

ΣN_n net induced yawing accelerations at $t = n$

p rolling velocity, radians per second except as otherwise indicated

r yawing velocity, radians per second except as otherwise indicated

ϕ angle of roll, radians except as otherwise indicated

ρ air density, slugs per cubic feet

V velocity along X-axis, feet per second

v sideslipping component of velocity, feet per second

q dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$

S wing area, square feet

m mass of airplane, slugs

k_X radius of gyration about X-axis, feet

k_Z radius of gyration about Z-axis, feet

t time, seconds

g gravitational acceleration $\left(32.2 \text{ ft/sec}^2\right)$

K_o, K_f, K_1, K_2, K_3 constants used in determining N_r

The subscripts n and $n - 1$ denote values corresponding to the time t and to the immediately preceding time $t - \Delta t$, respectively.

PROCEDURE FOR COMPUTING LATERAL MOTIONS BY STEP-BY-STEP INTEGRATION

All the procedures considered for determination of disturbed motions are based upon the following well-known dynamic lateral-motion equations for level flight:

$$\frac{dp}{dt} = \delta L_\delta + pL_p + rL_r + \beta L_\beta \qquad (1)$$

$$\frac{dr}{dt} = \delta N_\delta + pN_p + rN_r + \beta N_\beta \tag{2}$$

$$\frac{dv}{dt} = g \sin \phi - rV + \beta Y_\beta \tag{3}$$

$$\frac{d\phi}{dt} = p \tag{4}$$

$$\beta = \frac{v}{V} \tag{5}$$

The individual terms in equations (1) and (2) represent the values of the instantaneous angular accelerations produced by the magnitude of the aerodynamic moments acting on the airplane at any given instant of time. The individual terms in equation (3) similarly represent the instantaneous lateral accelerations produced by the gravitational and aerodynamic forces. The instantaneous accelerations are independent of the manner in which the aerodynamic moments and forces vary and are dependent only upon the instantaneous magnitudes of the moments and forces acting at any given time.

For the linear case, the acceleration terms such as βN_β and pN_p may be expressed as products of an angular displacement or velocity, as the case may be, and a constant slope representing the acceleration caused by the disturbance per unit disturbed motion. Equations (1) to (4) may therefore be directly integrated (reference 2).

For the nonlinear case, direct integration is seldom possible. When direct integration is not possible, the accelerations, such as βN_β, βY_β, and δL_δ, determined from model experimental data, may be plotted as functions of β; such a plot permits a solution for the nonlinear case of disturbed motions by the use of step-by-step integration or, when available, a differential analyzer. No variation of δL_δ and δN_δ with β was considered for the airplane in the present report since no such experimental data were available.

The appendix presents the data, the references, the calculations, and the information for curves such as figure 2 necessary for the formal step-by-step integration. The expression for N_r, as given in the appendix and used in conjunction with the method of the present report, differs slightly from the expression given in reference 6 in that the first term of the equation for the determination of C_{n_r} in reference 6

$$114.6\frac{l}{b}\left(C_{n_{\psi_{\text{tail on}}}} - C_{n_{\psi_{\text{tail off}}}}\right)$$

which represents the damping of the vertical tail and is suitable for propeller-off conditions, has been replaced herein by the expression

$$-114.6\frac{l}{b}\, C_{n_{\delta_r}} \delta_{r_{\alpha_v}}$$

Analysis indicated that the rotation of the propeller slipstream and sidewash in model tests precluded a reliable determination of the vertical-tail effectiveness $\frac{\partial C_n}{\partial \alpha_v}$ when the expression of reference 6 was used. The expression given in the present report is more general and is suitable for any power and propeller arrangement.

The values of K_f, K_2, and K_3 have not been solved for in the appendix since they are used for flaps-deflected conditions and the airplane used in the present report was in the cruising configuration. After the calculations indicated in the appendix have been made and after curves such as figure 2 have been plotted, the step-by-step integration form shown as table I may be used. In using the step-by-step integration, it may be desirable to use time increments of 1/10 second for computational convenience as well as for brevity of the solution combined with a fairly good degree of accuracy.

The integration indicated in table I is based upon the summation process of solution of equations (1) to (5). This summation process, as used in table I, may be expressed as

$$p_n = \left(\frac{dp}{dt}\right)_{n-1} \Delta t + p_{n-1} \qquad (6)$$

$$\phi_n = \frac{p_n + p_{n-1}}{2} \Delta t + \phi_{n-1} \tag{7}$$

$$r_n = \left(\frac{dr}{dt}\right)_{n-1} \Delta t + r_{n-1} \tag{8}$$

$$\beta_n = \left(\frac{d\beta}{dt}\right)_{n-1} \Delta t + \beta_{n-1} \tag{9}$$

where

$$\left(\frac{dp}{dt}\right)_{n-1} = \delta L_\delta + p_{n-1}L_p + r_{n-1}L_r + \left(\beta L_\beta\right)_{n-1} \tag{10}$$

$$\left(\frac{dr}{dt}\right)_{n-1} = \delta N_\delta + p_{n-1}N_p + r_{n-1}N_r + \left(\beta N_\beta\right)_{n-1} \tag{11}$$

$$\left(\frac{d\beta}{dt}\right)_{n-1} = \frac{g}{V} \sin \phi_{n-1} - r_{n-1} + \frac{\left(\beta Y_\beta\right)_{n-1}}{V} \tag{12}$$

The subscripts n and $n - 1$ denote values corresponding to the time t and to the immediately preceding time $t - \Delta t$, respectively.

The first step in using the step-by-step integration involves the insertion of values for the constant accelerations and derivatives δL_δ, δN_δ, L_p, L_r, N_p, and N_r in columns (3), (11), (20), (21), (24), and (25) in the underlined spaces provided in the headings of table I. The values in radian measure of the initial rate of roll p, the angle of bank ϕ, the rate of yaw r, and the angle of sideslip β should be inserted in columns (5), (8), (13), and (18) for $t = 0$. From curves such as figure 2, the values of βY_β, βL_β, and βN_β should be determined for the value of β at $t = 0$ ($\beta = 0$ in the present case). These values should be inserted in columns (14), (19), and (23) for $t = 0$.

Columns (9), (10), (15), (16), (20) to (22), and (24) to (26) may now be filled in for $t = 0$. Column (22) provides the induced rolling accelerations; column (26) provides the induced yawing accelerations. The net instantaneous rolling and yawing accelerations may now be determined for $t = 0$ by performing the computations indicated in columns (3) and (11).

By repeating the procedure indicated in the headings of table I and by using the sample values obtained for $t = 0$, the values of p, ϕ, r, and β are obtained for $t = 0.1$ second. After the value of β for $t = 0.1$ second has been obtained, corresponding values of βY_{β}, βL_{β}, and βN_{β} are determined and inserted in columns (14), (19), and (23) for $t = 0.1$ second. The net induced accelerations ΣL_n and ΣN_n for $t = 0.1$ second may now be determined (columns (22) and (26)) and, as a result, the values in columns (3) and (11) may be determined for $t = 0.1$ second. The remainder of table I for the other values of t may now be solved by repeating the procedure indicated in the headings and by using curves similar to figure 2.

The angle of bank ϕ was determined by averaging the rate of roll p (columns (5) to (7)). This averaging was not followed through for $\sin \phi$ and for r in the determination of β, because it was thought desirable to maintain simplicity in the table and the errors introduced by a disregard of these averages are small and are within the accuracy of the data used for the calculations in the appendix.

The step-by-step integration presented herein is not limited to the solution of motions produced by ailerons. Such integration may just as readily be applied to the solution of disturbed motions produced by rudder movements or by a combination of rudder and aileron movement. For the case of lateral disturbances caused by rudder alone, $\delta_a L_{\delta_a}$ and $\delta_a N_{\delta_a}$ would be changed to $\delta_r L_{\delta_r}$ and $\delta_r N_{\delta_r}$.

When the step-by-step integration is applied with variable derivatives to flight conditions involving accelerations greater than 1 g, the value of the airplane speed used should be the true airspeed V. The acceleration, however, must be considered in determining the airplane lift coefficient. The values of C_{n_a} and C_{l_a} (if an aileron movement is concerned) and the derivatives correspond to the new lift coefficient.

COMPARISON OF PROCEDURES FOR COMPUTING LATERAL DISTURBANCES

The characteristic curves obtained by the step-by-step integration are compared in figures 3 to 6 with the results obtained from actual flight tests; with the method of differential operators (reference 2), which is an exact solution dealing with constant slopes; and with an approximate analytical solution in which constant slopes are also used (reference 4) and which is applicable only to the solution of the sideslip angle. When the maximum sideslip angle was computed by the approximate method of reference 5, the computed value was found to be 57.5°, which does not compare with the $18\frac{3}{4}°$ determined from flight tests. When the value of C_n was considered equal to $C_{n_a} + C_{n_p}\frac{pb}{2V}$, the computed value of the maximum sideslip angle was determined to be 49.2°, which is still rather high. The present procedure provides the most accurate correlation with flight test results for all the motions considered.

It should be noted that the refinement used in the present report for the determination of N_r was not used in the application of the methods of references 2 and 4. It should also be noted that v/V, which is considered equal to the value of β in radians in all the procedures, is in its strictest sense equal to $\tan \beta$. The assumption that $\beta = \frac{v}{V}$ leads to much larger errors for large values than for small values of β. For example, consideration of these two sources of error reduces the maximum sideslip angle of 92°, shown for the approximate procedure of reference 4, to a value of 56°. The improved method in considering N_r accounted for 9°, whereas the other 27° were accounted for by the fact that v/V was considered equal to $\tan \beta$. In the case of the step-by-step procedure of the present report, the maximum sideslip angle would have been equal to about 25° if N_r had been determined by the method of reference 6. If v/V had been considered equal to $\tan \beta$, the maximum sideslip angle by the step-by-step method would have been reduced about $\frac{3}{4}°$.

For solutions involving the assumption of linear slopes, the slopes used in the present problem were arbitrarily measured through $\psi = 0^{\circ}$. If the more usual practice of selecting the average slopes over a wider range of yaw angles had been employed, the calculated results would have approached more closely the results of the variable-slope method. For cases in which vertical tail loads in high-speed dives are of primary concern, however, small angles of sideslip may be critical, and consideration of average slopes over a wide range of yaw angles may be unwise. It appears therefore that, although the previous procedures may be reasonably reliable in a number of instances in which the characteristic C_l, C_n, and C_Y curves possess approximately linear relationships, nonlinear characteristics occur with sufficient frequency to make the general use of the nonlinear step-by-step procedure desirable.

In order to determine the importance of the lateral-acceleration term βY_{β}, the present procedure was repeated with $\beta Y_{\beta} = 0$. Although the resulting curves indicate that the influence of βY_{β} for the subject airplane was not very large, the effect of βY_{β} may be more significant for other types of airplane and therefore should not be neglected.

A comparison of the step-by-step solution using constant slopes with the method of differential operators (reference 2) indicated that values obtained by the step-by-step solution tended to deviate a little more from flight test results than the values obtained by the operational method. The step-by-step solution, for this particular comparison, apparently gives a sideslip angle approximately 2° larger than the operational procedure of reference 2. The tendency of the step-by-step solution in the linear case to deviate a little more from flight tests than a direct integration procedure may reasonably be presumed to persist in the application of the step-by-step solution to the nonlinear case, as in the present report. Further refinement of the step-by-step procedure may therefore be expected to provide correspondingly closer agreement with flight. The Runge-Kutta summation method (references 8 and 9) provides such refinements of procedures. The step-by-step procedure as outlined in the present report, however, is believed to provide sufficient engineering accuracy when no more than the first quarter-cycle of the motion is required.

Although δL_{δ} and δN_{δ} were considered constants in the preceding example, further analysis indicated that the rolling and yawing accelerations resulting from aileron deflection should also be considered functions of β for a greater degree of accuracy. It is quite possible that L_p, N_p, L_r, and N_r are not constant as ordinarily assumed and as assumed in the present report. If these parameters are not constant, some of the discrepancy that still exists between flight test results and the present method would be explained. Until experimental data from dynamic-model tests are available, however, these values must be presumed constant for lack of more complete information. Other possible sources of discrepancy between calculations and flight results are the assumptions of level flight, constant normal acceleration, constant speed, and instantaneous control deflection. For practical purposes, however, it was not believed necessary to take these factors into account.

CONCLUSIONS

A procedure based upon step-by-step integration is presented for determining the disturbed motions of an airplane resulting from the deflection of the lateral or directional controls for the case of nonlinear derivatives. A comparison of the step-by-step procedure with other methods indicated the following conclusions:

1. The calculated disturbed motions of an airplane resulting from abrupt control movement will be in better agreement with the results obtained from flight tests if the variation of the experimentally determined rolling, yawing, and sideslipping accelerations βL_{β}, βN_{β}, and βY_{β} with the angle of sideslip β is considered. The sideslipping acceleration βY_{β}, which is often assumed negligible, should be considered. The variation of the rolling and yawing accelerations $\delta_a L_{\delta_a}$ and $\delta_a N_{\delta_a}$ resulting from aileron movement probably should also be considered when sufficient data are available. The variation of the dynamic derivatives L_p, N_p, L_r, and N_r should also be taken into account when sufficient dynamic-test data are available.

2. The value of the maximum sideslip angle for use in the determination of the vertical-tail loads in rolling pull-out maneuvers should be obtained by using the step-by-step integration method.

3. The step-by-step integration may be applied to the solution of motions produced by rudder movements or by a combination rudder and aileron movement, as well as to the solution of motions produced by ailerons alone when only the first quarter-cycle of the motion is desired.

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APPENDIX

DETERMINATION OF DYNAMIC LATERAL MOTIONS OF A FIGHTER AIRPLANE DUE TO ABRUPT AILERON MOVEMENT

Data required.- For the fighter airplane used in the illustration, the data required for the determination of dynamic lateral motions resulting from abrupt aileron movement are as follows:

b, ft.	42.83	$C_{d_{o_w}}$	0.01
b_f, percent b	66	$\Delta C_{d_{o_f}}$	0
λ	0.50	C_{l_a}	0.04
A	5.5	C_{n_a}	-0.0065
l, ft	20.5	$C_{n_{\delta_r}}\delta_{r_{\alpha_v}}$	0.001474
δ_{a_L}, deg	12.75	V, fps	142.2
δ_{a_R}, deg	-17	q, lb/sq ft	24.09
δ_f, deg	0	m, slugs	358
α_a, deg	17.1	S, sq ft	334
C_{L_w}	1.42	$k_X{}^2$, sq ft	33.52
ΔC_{L_f}	0	$k_Z{}^2$, sq ft	57.9

Landing gear . . . . . . Retracted

The value of $\delta_{r_{\alpha_v}} = -2.0$ is determined from reference 10.

Procedure.- From reference 3, determine

$$C_{l_p} = -0.425$$

$$C_{n_p} = -0.0655$$

$$C_{l_r} = 0.308$$

Then, from reference 6, determine

$$K_o = -0.33 \frac{1 + 3\lambda}{2 + 2\lambda}$$

$$= -0.2749$$

From reference 7, determine

$$K_1 = -0.0202$$

Compute the following:

$$L_p = C_{l_p} \frac{b}{2V} \frac{qSb}{mk_X^2}$$

$$= -1.834$$

$$\delta_a L_{\delta_a} = C_{l_a} \frac{qSb}{mk_X^2}$$

$$= 1.1496$$

$$N_p = C_{n_p} \frac{b}{2V} \frac{qSb}{mk_Z^2}$$

$$= -0.161$$

$$\delta_a N_{\delta_a} = C_{n_a} \frac{qSb}{mk_Z^2}$$

$$= -0.108$$

$$L_r = C_{l_r} \frac{b}{2V} \frac{qSb}{mk_X^2}$$

$$= 1.33$$

$$N_r = \left[-114.6 \frac{l}{b} C_{n_{\delta_r}} \delta_{r_{a_v}} + K_o C_{d_{o_W}} + K_f \Delta C_{d_{o_f}} + K_1 C_{L_W}{}^2 + K_2 \Delta C_{L_f} C_{L_W} + K_3 \left(\Delta C_{L_f}\right)^2\right] \frac{qSb}{mk_Z^2} \frac{b}{2V}$$

$$= -0.3108$$

From wind-tunnel data for the configuration considered (fig. 1), plot the following against β or $-\psi$:

$$\beta L_\beta = C_l \frac{qSb}{mk_X^2}$$

$$\beta N_\beta = C_n \frac{qSb}{mk_Z^2}$$

$$\beta Y_\beta = C_Y \frac{qS}{m}$$

The values of K_f, K_2, and K_3 have not been solved for since they are used for flaps-deflected conditions and the airplane used in the present report was in the cruising configuration. For flaps-deflected conditions, K_f may be determined from the following formula from reference 6:

$$K_f = -0.33\left(\frac{b_f}{b}\right)^3 \frac{4 - 3\frac{b_f}{b}(1 - \lambda)}{2 + 2\lambda}$$

The values of K_2 and K_3 may be determined from reference 7.

Although the values of C_{n_p} and C_{l_r} in the present report have been determined solely from the curves of reference 3, it may be desirable in some cases to include the effects of the vertical tail by use of the method of reference 11.

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TABLE I.- PORTION OF TABLE USED TO DETERMINE DISTURBED MOTIONS OF A FIGHTER AIRPLANE RESULTING FROM ABRUPT DEFLECTION OF AILERONS

[Accelerations βL_β, βN_β, and βY_β determined from experimental data as functions of β]

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
t (sec)	Δt (sec)	$\left(\frac{dp}{dt}\right)_n$	Δp	p (radians/sec)	$\bar{p}$	$\Delta\phi$	ϕ (radians)	$\sin\phi_n$	$\frac{g}{V}\sin\phi_n$	$\left(\frac{dr}{dt}\right)_n$	Δr	r (radians/sec)	$\left(\beta Y_\beta\right)_n$	$\frac{\left(\beta Y_\beta\right)_n}{V}$
		$\delta L_\delta + \Sigma L_n$	$\left(\frac{dp}{dt}\right)_n \Delta t$	$\Delta p + p_{n-1}$	$\frac{p_n + p_{n-1}}{2}$	$\bar{p}\,\Delta t$	$\Delta\phi + \phi_{n-1}$			$\delta N_\delta + \Sigma N_n$	$\left(\frac{dr}{dt}\right)_n \Delta t$	$\Delta r + r_{n-1}$		
		1.1496+(22)	(3)×(2)	$p_0+\Sigma(4)$	$\frac{(5)_n+(5)_{n-1}}{2}$	(6)×(2)	$\phi_0+\Sigma(7)$	sin (8)	$\frac{g}{V}$×(9)	-0.108+(26)	(11)×(2)	$r_0+\Sigma(12)$	From curves	(14)/V
0		1.1496		0			0	0	0	-0.108		0	0	0
	0.10		0.11496		0.05748	0.005748					-0.0108			
.1		.9244		.11496			.005748	.00575	.00130	-.1232		-.0108	0	0
	.10		.09244		.16118	.01612					-.01232			
.2		.7335		.20740			.02187	.02181	.00494	-.1322		-.02312	-.015	-.00010
	.10		.07335		.24408	.02441					-.01322			
.3		.5714		.28075			.04628	.04623	.01046	-.1389		-.03634	-.016	-.00011
	.10		.05714		.30932	.03093					-.01389			
.4		.4311		.33789			.07721	.07707	.01745	-.1418		-.05023	-.017	-.00049
	.10		.04311		.35944	.03594					-.01418			
.5		.3021		.3800			.11315	.11286	.02555	-.1403		-.06441	-.120	-.00084
	.10		.03021		.39610	.03961					-.01403			
.6		.1831		.41121			.15276	.15212	.03444	-.1379		-.07844	-.215	-.00151

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(1)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
t (sec)	$\frac{g}{V}\sin\phi_n - r_n + \frac{\left(\beta Y_\beta\right)_n}{V}$	$\Delta\beta$	β (radians)	$\Sigma L_n = p_n L_p + r_n L_r + \left(\beta L_\beta\right)_n$				$\Sigma N_n = p_n N_p + r_n N_r + \left(\beta N_\beta\right)_n$				Summary			
			$\Delta\beta + \beta_{n-1}$	$\left(\beta L_\beta\right)_n$	$p_n L_p$	$r_n L_r$	ΣL_n	$\left(\beta N_\beta\right)_n$	$p_n N_p$	$r_n N_r$	ΣN_n	p_n (deg/sec)	ϕ_n (deg)	r_n (deg/sec)	β_n (deg)
	(10)-(13)+(15)	(16)×(2)	$\beta_0+\Sigma(17)$	From curves	(5)×-1.834	(13)×1.33	(19)+(20)+(21)	From curves	(5)×-0.161	(13)×-0.31	(23)+(24)+(25)	57.3×(5)	57.3×(8)	57.3×(13)	57.3×(18)
0	0		0	0	0	0	0	0	0	0	0	0	0	0	0
		0													
.1	.01210		0	0	-.2108	-.0144	-.2252	0	-.0185	.0033	-.0152	6.59	.33	-.62	0
		.00121													
.2	.02796		.00121	-.005	-.3804	-.0307	-.4161	.002	-.0334	.0072	-.0242	11.58	1.25	-1.32	.07
		.002796													
.3	.04669		.00401	-.015	-.5149	-.0483	-.5782	.003	-.0452	.0113	-.0309	16.09	2.62	-2.08	.23
		.004669													
.4	.06817		.00868	-.032	-.6197	-.0668	-.7185	.005	-.0544	.0156	-.0338	19.36	4.42	-2.88	.50
		.006817													
.5	.08912		.01554	-.063	-.6988	-.0857	-.8475	.009	-.0613	.0200	-.0323	21.83	6.48	-3.69	.89
		.008912													
.6	.11137		.02445	-.108	-.7542	-.1043	-.9665	.012	-.0662	.0243	-.0299	23.56	8.75	-4.49	1.40

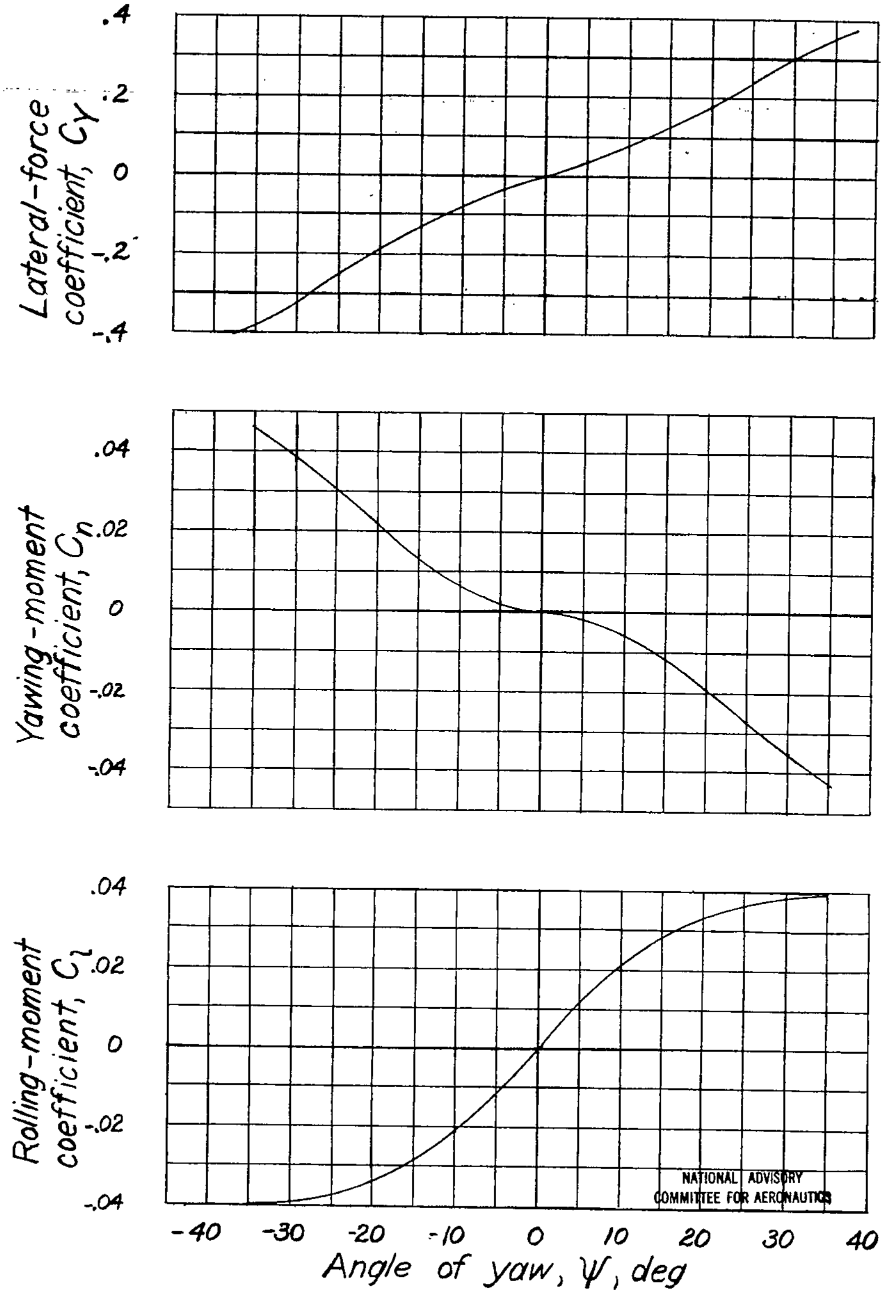

Figure 1.– Variation of the directional and lateral coefficients C_l, C_n, and C_Y with angle of yaw ψ determined from tests of a $\frac{1}{6}$-scale model of a fighter airplane in the Langley 7- by 10-foot tunnel. Cruising configuration; C_L=1.42; δ_r=0°; δ_a=0°.

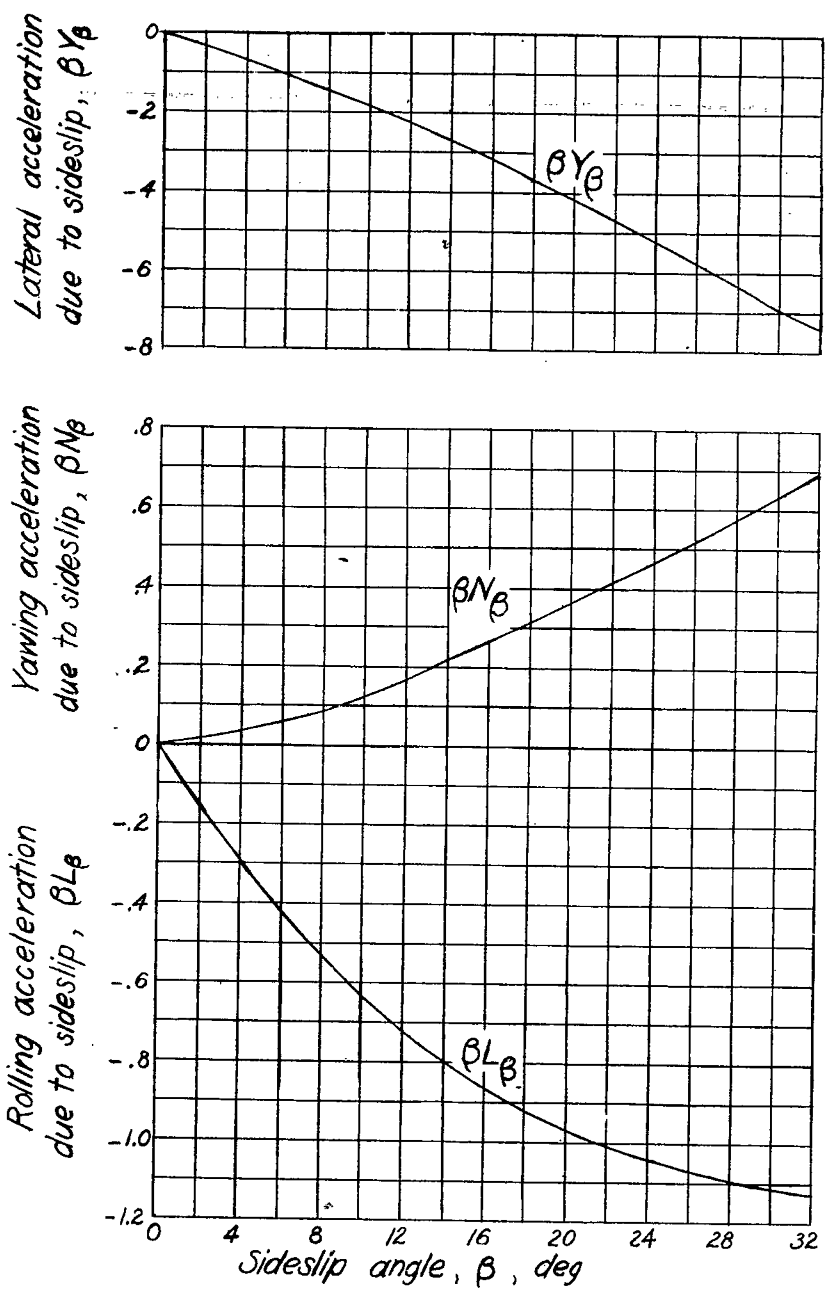

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Figure 2.– Curves of the accelerations βL_β, βN_β, and βY_β as functions of the sideslip angle β determined from tests of a $\frac{1}{6}$-scale model of a fighter airplane in the Langley 7-by 10-foot tunnel. $\delta_r = 0^\circ$; $\delta_a = 0^\circ$

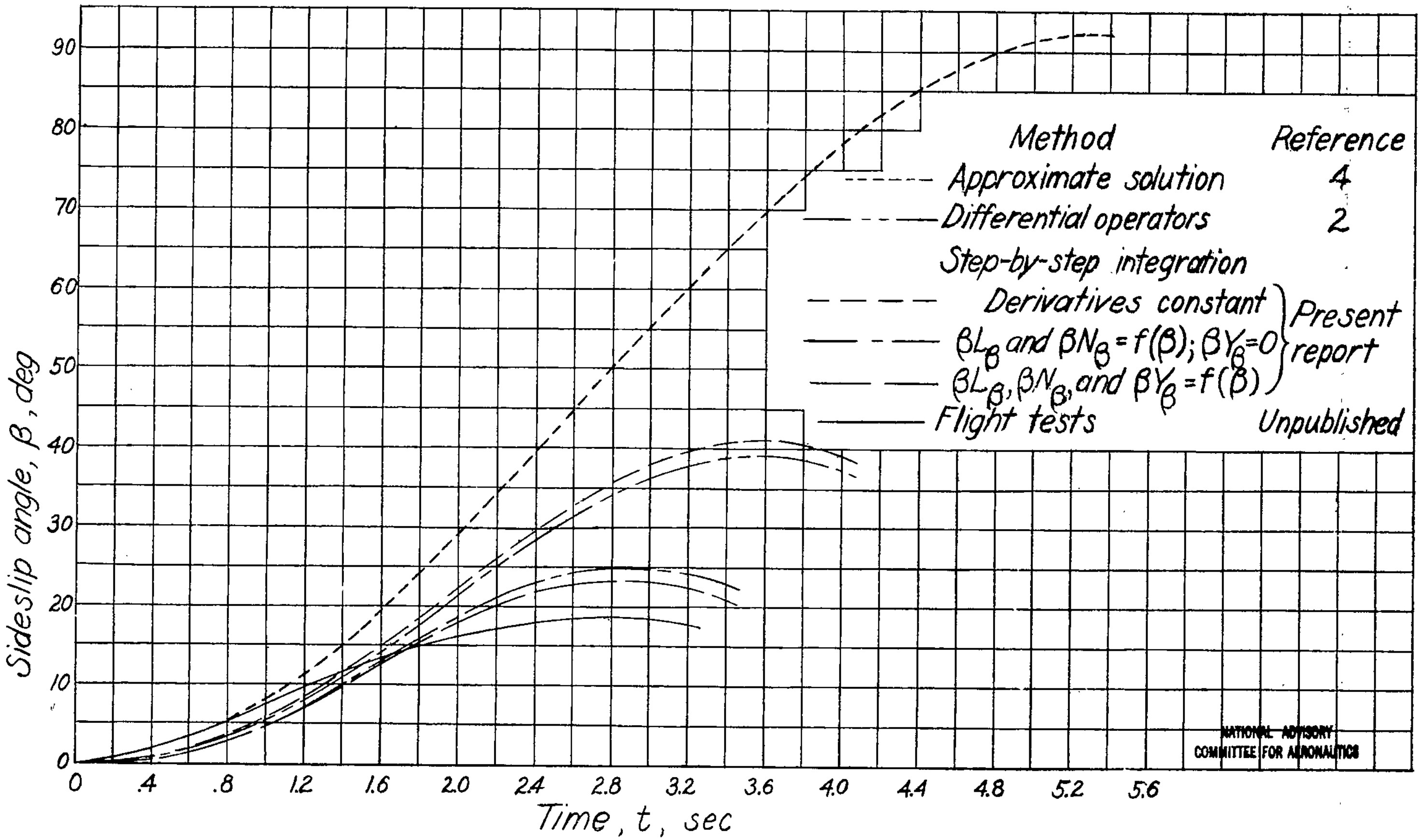

Figure 3.- Comparison of sideslip curves for a fighter airplane obtained by several different methods of calculation and by flight tests. Airplane in cruising configuration; $C_L = 1.42$.

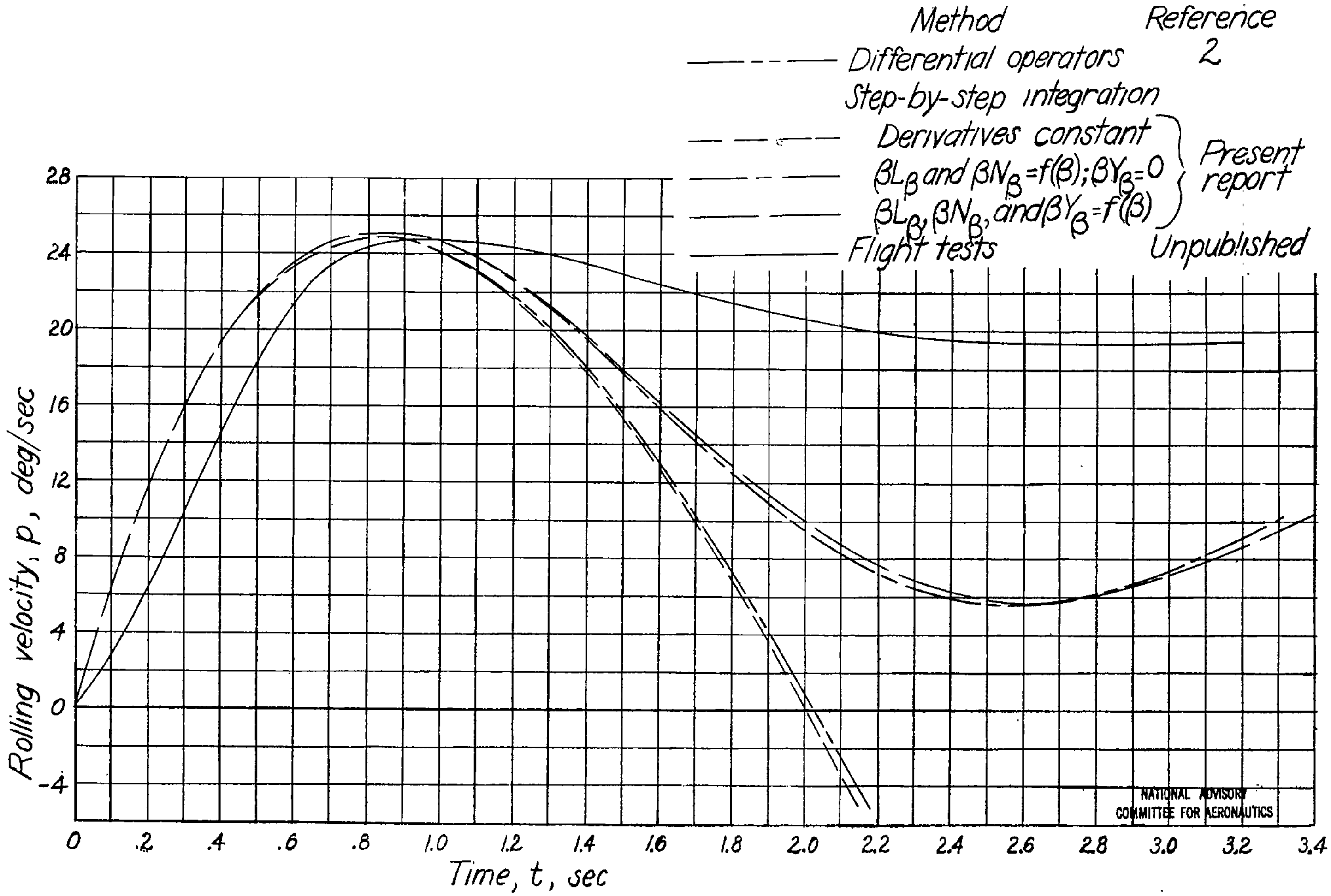

Figure 4.- Comparison of rolling-velocity curves for a fighter airplane obtained by several different methods of calculation and by flight tests. Airplane in cruising configuration; $C_L = 1.42$.

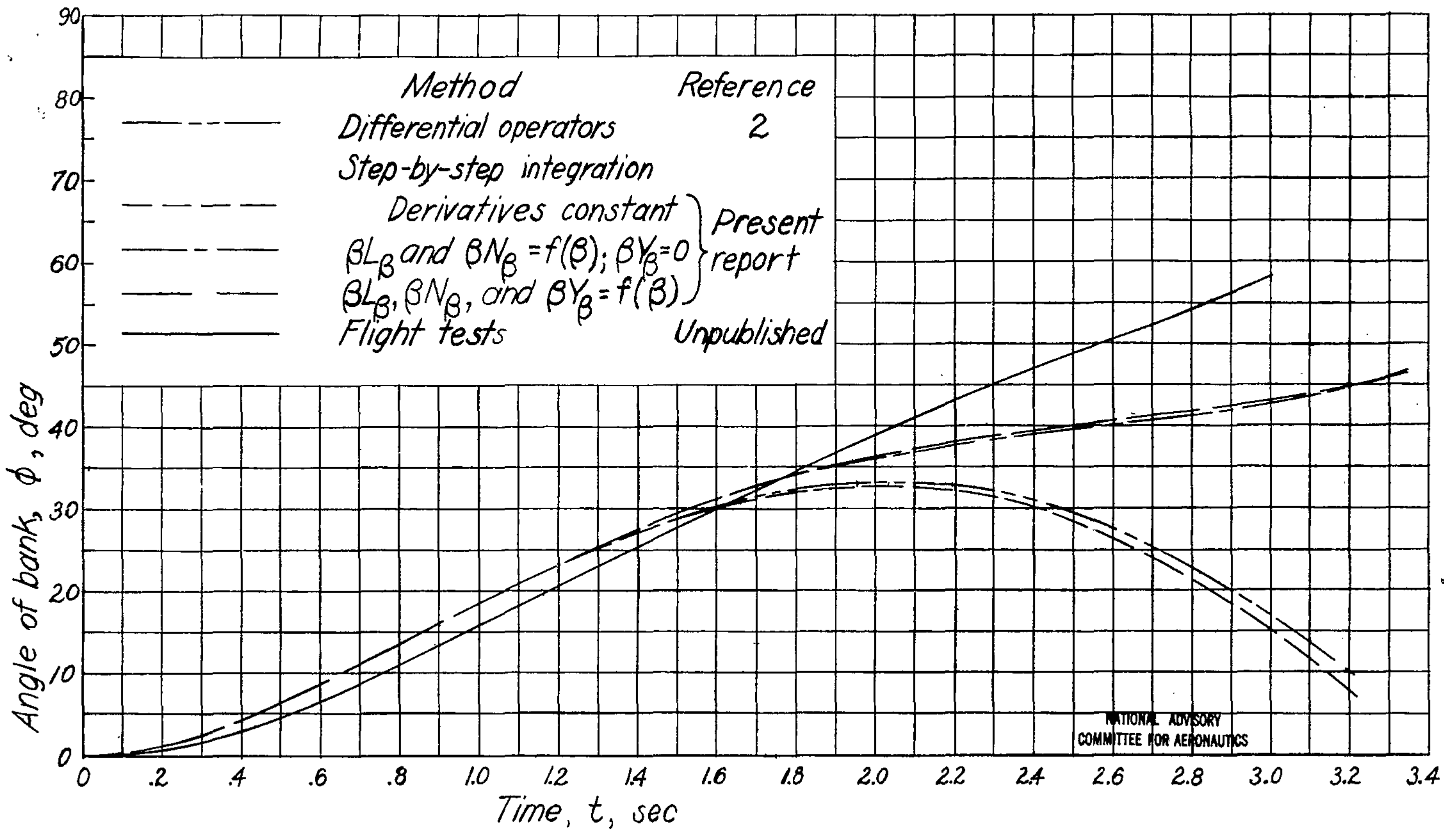

Figure 5.– Comparison of angle-of-bank curves for a fighter airplane obtained by several different methods of calculation and by flight tests. Airplane in cruising configuration; $C_L = 1.42$.

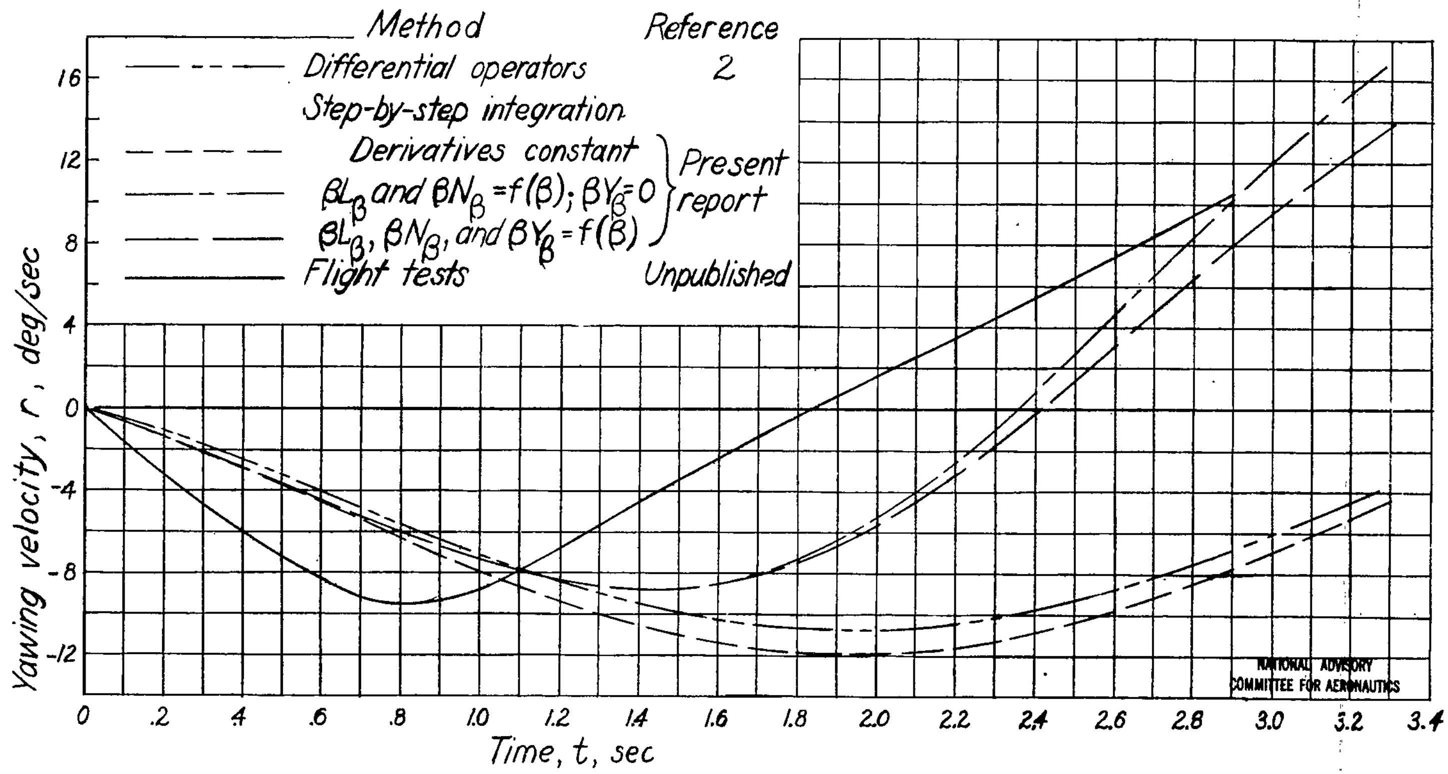

Figure 6.- Comparison of yawing-velocity curves for a fighter airplane obtained by several different methods of calculation and by flight tests. Airplane in cruising configuration; $C_L = 1.42$.